KHYBER PAKHTUNKHWA PUBLIC SERVICE COMMISSION
COMPETITIVE EXAMINATION FOR THE POSTS OF PMS-2022

APPLIED MATHEMATICS PAPER-II

Time Allowed: 03 Hours **Maximum Marks: 100**

Instructions: *Attempt total **FIVE** questions, **TWO** questions from Section-A, **ONE** question from Section-B and **TWO** questions from Section-C*

Section-A

1. (a) By using the method of undetermined coefficients, solve the following differential equation: $y'' + 4y = xe^x + x\sin 2x$ (10)

   (b) By the method of variation of parameters find the general solution of
   $x^2\frac{d^2y}{dx^2} - x(x+2)\frac{dy}{dx} + (x+2)y = x^3$
   given that $y = xe^x$ is a solution of the associated homogeneous equation
   $x^2\frac{d^2y}{dx^2} - x(x+2)\frac{dy}{dx} + (x+2)y = 0.$ (10)

2. (a) Solve $r = a^2t$, $\left(where\ r = \frac{\partial^2 z}{\partial x^2},\ and\ t = \frac{\partial^2 z}{\partial y^2}\right)$ by using Monge's method. (10)

   (b) Find a partial differential equation by eliminating a, b, c from $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.$ (10)

3. (a) Apply the method of separation of variables to find the solution of the two-dimensional Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$,
   along with the boundary conditions
   $u(0,y) = 0, \quad 0 \le y \le b, \quad u(a,y) = 0, \quad 0 \le y \le b$
   $u(x,0) = 0, \quad 0 \le x \le a, \quad u(x,b) = f(x), \quad 0 \le x \le a$
   where a, b are positive constants and f is a specified function of x. (10)

   (b) An insulated rod of length l has its ends A and B kept at a° celsius and b° celsius respectively until steady state conditions prevail. The temperature at each end is suddenly reduced to zero degree celsius and kept so. Find the resulting temperature at any point of the rod taking the end A as origin. (10)

Section-B

4. (a) Prove that the alternating symbol $\in_{ijk}$ is a Cartesian tensor of rank 3. (10)

   (b) If A_{ijk} and B_{mn} are two Cartesian tensors of rank 3 and 2 respectively, prove that $C_{ijkmn} = A_{ijk}B_{mn}$ is also a tensor of rank 5. (10)

5. (a) Prove that the form of a tensor equation remains the same in every rectangular coordinate system. (10)

   (b) Prove by using tensor method $\left[\nabla \times (\nabla \times \vec{A})\right]_i = \left[\nabla(\nabla \cdot \vec{A})\right]_i - (\nabla^2\vec{A})_i$
   for $i = 1,2,3$. (10)

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Section-C

6. (a) Use Newton Raphson's method to approximate the solution of the following equation

$x^{\frac{1}{5}} - 3.23\,x^4 - 5.54x + 9.84 = 0\,;\, x_0 = 0.1.$ (10)

(b) The solid of revolution obtained by rotating the region under the curve $y = f(x), \quad a \le x \le b$, about the $x - axis$ has surface area given by $area = 2\pi \int_a^b f(x)\sqrt{1 + [f'(x)]^2}\,dx$. Find the area of the function $f(x) = x^3,\ 0 \le x \le 1$, using Simpson's $\frac{1}{3}rd$ rule. Take $n = 10$ (Number of intervals). (10)

7. (a) The following table gives the coordinates of points on a certain polynomial

x	0	0.5	1	1.5	2
y	2.0286	2.4043	2.7637	3.1072	2.4350

Calculate the radius of curvature ρ(roh) using the following formula

$\rho = \frac{\left[1+(y')^2\right]^{1.5}}{y''}$, at the point $x = 0.25$. (10)

(b) The following table gives the values of density of saturated water for various temperatures of saturated steam.

$Temp°C\ (= T)$	100	150	200	250	300
$Density\ hg/m^3 (= d)$	958	917	865	799	712

Find by interpolation, the densities when the temperatures are $130°C$ and $275°C$ respectively. (10)

8. (a) Solve the system of equations

$$28x + 4y - z = 32$$
$$x + 3y + 10z = 24$$
$$2x + 17y + 4z = 35$$

By using Guass-Seidel method. Take $(x_0, y_0, z_0)^T = (0, 0, 0)^T$, using 5 significant digits in computation. (10)

(b) Interpolate the missing values in the following table:

x	1911	1912	1913	1914	1915	1916	1917	1918	1919
y	76.6	78.2	-------	77.7	78.7	-------	80.6	77.6	78.6

(10)