PUNJAB PUBLIC SERVICE COMMISSION

COMBINED COMPETITIVE EXAMINATION
FOR RECRUITMENT TO THE POSTS OF
PROVINCIAL MANAGEMENT SERVICE, ETC -2021
CASE NO. 3C2022

SUBJECT: **MATHEMATICS (PAPER-I)**

TIME ALLOWED: **THREE HOURS** **MAXIMUM MARKS: 100**

NOTE:

i. All the parts (if any) of each Question must be attempted at one place instead of at different places.
ii. Write Q. No. in the Answer Book in accordance with Q. No. in the Q. Paper.
iii. No Page/Space be left blank between the answers. All the blank pages of Answer Book must be crossed.
iv. Extra attempt of any question or any part of the question will not be considered.

NOTE: **Attempt FIVE Questions in All. THREE Questions from Section 'A' and TWO Questions from Section 'B'. Calculator is allowed. (Not programmable)**

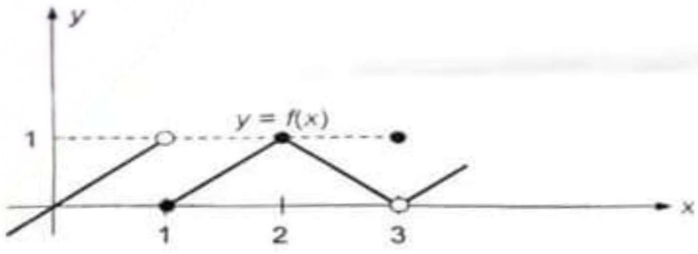
SECTION-A

Q.1 **(a)** For the function $f(x)$ graphed in the adjoining figure, find the following limits or explain why they do not exist.

(i) $\lim_{x \to 1} f(x)$ **(ii)** $\lim_{x \to 2} f(x)$

(iii) $\lim_{x \to 3} f(x)$

Also discuss the continuity of $f(x)$ at $x = 1$, $x = 2$ and $x = 3$.



(b) Differentiate $y = \sqrt[3]{\dfrac{x(x^2+1)}{(x-1)^2}}$ with respect to x. **(10 + 10 = 20 Marks)**

Q.2 **(a)** For what values of a, m, and b does the function

$$f(x) = \begin{cases} 3, & x = 0 \\ -x^3 + 3x + a, & 0 < x < 1 \\ mx + b, & 1 \le x \le 2 \end{cases}$$

satisfy the hypotheses of mean value theorem on the interval $[0, 2]$?

(b) A box with rectangular base, whose length is twice its width, is to have a closed top. The area of the material in the box is to be 192 in². What should the dimensions of the box be in order to have the largest possible volume?

(10 + 10 = 20 Marks)

Q.3 **(a)** Show that $\displaystyle\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}}\,dx = \frac{\pi}{4}$.

(b) Evaluate $\displaystyle I = \int_0^4 \int_0^{4-x} \int_0^{4-x-y} dz\,dy\,dx$. **(10 + 10 = 20 Marks)**

Q.4 **(a)** A plot of land lies between a straight fence and a curved stream at distance x meters from one end of the fence, the width y meters of the plot was measured as follows

x	0	10	20	30	40	50	60	70	80
y	0	32	44	58	63	50	30	32	0

Find the approximate area of the plot by using trapezoidal rule.

(b) Solve the differential equation $(x+1)\frac{dy}{dx} - ny = e^x (x+1)^{n+1}$.

(10 + 10 = 20 Marks)

Q.5 **(a)** Assume that the half life of the radium in a piece of lead is 1500 years. How much radium will remain in the lead after 2500 years?

(b) A particle of mass m is moving under the action of the forces $F_1 = -m\omega^2 x$, $F_2 = mF_0 t$, $F_3 = -2m\mu\frac{dx}{dt}$. Assuming that damping is small, set up and solve the equation of motion.

(10 + 10 = 20 Marks)

SECTION-B

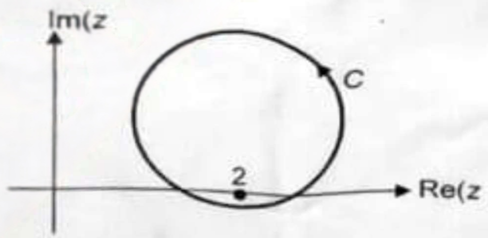
Q.6 **(a)** Find radius and interval of convergence of the series of $\sum_{1}^{\infty} \frac{n!\, x^n}{(2n)!}$.

(b) Prove that $(\sin x + i\cos x)^n = \cos n\left(\frac{\pi}{2} - x\right) + i\sin n\left(\frac{\pi}{2} - x\right), n \in Z$.

(10 + 10 = 20 Marks)

Q.7 **(a)** Construct the analytic function whose real part is $e^{-x}[(x^2 - y^2)\cos y + 2xy\sin y]$.

(b) Compute $\int_C \frac{z^2+2}{z(z^2-4)(z+4)}dz$, where C is the curve shown in the figure below:



(10+10=20 Marks)

Q.8 **(a)** Find the tangent and normal to the curve $x^2 - xy + y^2 = 7$ at the point $(-1, 2)$.

(b) Find the curvature and torsion of the circular helix $\vec{r} = (a\cos u,\ a\sin u,\ bu)$.

(10 + 10 = 20 Marks)