ROLL NO

PUNJAB PUBLIC SERVICE COMMISSION

COMBINED COMPETITIVE EXAMINATION FOR RECRUITMENT TO THE POSTS OF PROVINCIAL MANAGEMENT SERVICE, ETC -2023

CASE NO. 1C2024

SUBJECT: MATHEMATICS (PAPER-I)

TIME ALLOWED: THREE HOURS **MAXIMUM MARKS: 100**

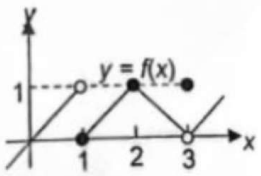
NOTE:

i. All the parts (if any) of each Question must be attempted at one place instead of at different places.
ii. Write Q. No. in the Answer Book in accordance with Q. No. in the Q. Paper.
iii. No Page/Space be left blank between the answers. All the blank pages of Answer Book must be crossed.
iv. Extra attempt of any question or any part of the question will not be considered.

NOTE: **Attempt FIVE Questions in All including THREE questions from Part-A and TWO questions from Part-B. Calculator is allowed. (Non-Programmable)**

PART-A

Q.No.1 **(a)** For the function $f(x)$ graphed here, find the following limits or explain why they do not exist.



(i) $\lim_{x\to 1} f(x)$, (ii) $\lim_{x\to 2} f(x)$,

(iii) $\lim_{x\to 2.5} f(x)$, (iv) $\lim_{x\to 2.5} f(x)$

(b) Find the values of a and b that make the following function differentiable for all x-values.

$$f(x)=\begin{cases} ax+b, & x>-1 \\ bx^2-3, & x\le -1 \end{cases}$$

(10+10=20 Marks)

Q.No.2 **(a)** Discuss the validity of Rolle's theorem of $f(x)=x(x+3)e^{-\frac{x}{2}}$ on $[-3,0]$. Find 'c' (if possible).

(b) Use Mean Value Theorem to show that $|\sin x-\sin y|\le|x-y|$ for any real numbers x, y.

(10+10=20 Marks)

Q.No.3 **(a)** Find the area of region bounded by the curve $y=x^2-4x$, the x-axis, and the lines $x=1$ and $x=3$.

(b) Using rectangular rule for $n=5$, approximate the value of the definite integral $\int_0^1 \frac{dx}{1+x^3}$.

(10+10=20 Marks)

Q.No.4 **(a)** Find the volume of the tetrahedron bounded by the coordinate planes and the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$, where a, b, c are positive.

(b) The area in the first quadrant bounded by the parabola $y^2=4ax$ and its latus rectum is revolved about the x-axis. Find the volume of the solid generated.

(10+10=20 Marks)

Q.No.5 **(a)** Solve $x\frac{dy}{dx}+y=y^2\ln x$.

(b) An oscillator moves under the forces
restorative force $=-kx$
damping force $=-2\mu\dot{x}$
driving force $=F_0e^{-at}$ each force being per unit mass.
Set up and solve the equation of motion completely. **(10+10=20 Marks)**

Q.No.6 **(a)** Prove that the function $f(z) = \sqrt{|xy|}$ is not differentiable at origin although Cauchy-Riemann conditions are satisfied at origin.

(b) Evaluate $\int_C \frac{z}{(z-1)(z+2i)} dz$ where (i) $C: |z| = \frac{1}{2}$, (ii) $C: |z| = \frac{3}{2}$.

(10+10=20 Marks)

Q.No.7 **(a)** Transform $x^2 + y^2 - z = 9$ into spherical coordinates.

(b) The tangent at any point on the curve $x^3 + y^3 = 2a^3$ makes intercepts p and q on the coordinate axes respectively. Show that $p^{-3/2} + q^{-3/2} = 2^{-1/2} a^{-3/2}$

(10+10=20 Marks)

Q.No.8 **(a)** Find the tangent line and normal plane to the curve $\vec{x} = t\,\hat{e}_1 + t^2\,\hat{e}_2 + t^3\,\hat{e}_3$ at $t = 1$.

(b) Find the curvature and torsion of $\vec{r} = (a\cos\theta,\ a\sin\theta,\ a\theta\cot\alpha)$.

(10+10=20 Marks)