KHYBER PAKHTUNKHWA, PUBLIC SERVICE COMMISSION

COMPETITIVE EXAMINATION FOR THE POSTS OF PROVINCIAL MANAGEMENT SERVICES (BPS-17).

2013

PURE MATHEMATICS, PAPER-I

TIME ALLOWED: 03 HOURS **MAX:MARKS: 100**

Note: Attempt five questions in all, selecting at least two questions from each section. Extra attempt of a question or part will not be considered. All questions carry equal marks.

Section-A

Q1(a). Define a cyclic group and Show that every subgroup of a cyclic group is cyclic. (10)

(b) Let $\mathbb{Z}$ be the group of integers under addition. Let $G = \left\{ \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}, n \in \mathbb{Z} \right\}$. Show that G is a group under matrix multiplication and that $\mathbb{Z} \cong G$. (10)

Q 2(a). Let H be a subgroup of a finite group G. Then Show that order of H divides the order of G. (10)

(b) Let $\alpha : G \longrightarrow G_1$ be a group homomorphism. Let $K = \text{Ker}(\alpha)$. Then Show that $\alpha(G)$ is isomorphic to $G/\text{Ker}(\alpha)$. (10)

Q3(a) Let A be an ideal of a ring R. Then Show that A is maximal in R if and only if R/A is a simple ring. (10)

(b) Let W be a subspace of a finite-dimensional vector space V, then Show that W is finite dimensional, $\dim W \leq \dim V$ and $\dim(V/W) = \dim V - \dim W$ (10)

Q4(a) Solve the following system of equations:

$$x+y+z = 1$$
$$2x+3y+4z = 1$$
$$x-y-z = 0 \quad (10)$$

(b) Let R be a commutative ring with identity, whose only ideals are $\{0\}$ and R itself, then Show that R

Section B

Q5(a) Find the equations of the straight line passing through the point $(6,-2,3)$ and perpendicular to the yz-plane. (10)

(b) Determine whether the following pair of lines intersect or not. If they intersect, then find their point of intersection.

$$\frac{x+3}{2} = \frac{y}{-2} = \frac{z-7}{6} \quad ; \quad \frac{x+6}{1} = \frac{y+5}{-3} = \frac{z-1}{2} \quad (10)$$

Q6(a) Find the equation of the plane passing through the line of intersection of the planes

$$2x - y + 3z = 0 \quad , \quad x + 2y - 2z = 3$$

and which is perpendicular to the xz-plane. (10)

(b) The points $A(3,2,-4)$, $B(-1,1,-2)$, $C(-2,3,3)$ and $D(-3,-2,1)$ are corners of a tetrahedron. Find the shortest distance between AC and BD. (10)

Q7(a) Find the total arc length of the cardioid

$$r = 4 + 4\cos\theta \,. \quad (10 \text{ points})$$

(b) A point P has rectangular coordinates $(-1,1,0)$. Find its cylindrical coordinates as well as spherical coordinates. (10)

Q8(a) The endpoints of a diameter of a sphere are $(3,1,-2)$ and $(5,7,-4)$. Find its equation. (10 points)

(b) Find the angle between the tangents to the following curves at their point of intersection

$$r = \sqrt{2}\sin\theta \quad , \quad r^2 = \cos 2\theta \,. \quad (10)$$