

PUBLIC SERVICE COMMISSION, PESHWAR

COMPETITIVE EXAMINATION FOR PROVINCIAL MANAGEMENT SERVICES, 2018

PURE MATHEMATICS (PAPER-I)

TIME ALLOWED: Three Hours

Max. Marks: 100

Instruction: Attempt three questions from Section A and two questions from section B

SECTION A

Q1. (a) State and prove the first isomorphism theorem for groups. (10)

(b) Evaluate the normalizer and centralizer of  $Q_8$ . (10)

Q2. (a) Define subring, Integral domain and the relation between them. (10)

(b) Show that  $\alpha = \sqrt{1 + \sqrt[3]{2}}$  is algebraic over  $Q$  by finding  $f(x) \in Q[x]$  such that  $f(\alpha) = 0$ . (10)

Q3. (a) Prove the following results:

Let  $S$  be a non-empty set of vectors in a vector space  $V$  over a field  $F$ . Then  $\langle S \rangle$  is a subspace of  $V$  containing  $S$  and it is the smallest subspace of  $V$  containing  $S$ . (10)

(b). Determine a basis for the null space of matrix. (10)

$$\begin{bmatrix} 1 & 2 & -4 & 2 & -3 \\ 2 & -2 & 1 & -2 & 0 \\ 0 & 1 & -1 & 1 & -1 \\ 1 & 3 & 5 & -6 & -3 \end{bmatrix}$$

Q4. (a) Solve the system of equations. (10)

$$2x + y - 2z = 10$$

$$3x + 2y + 2z = 1$$

$$5x + 4y + 3z = 4$$

(b) Find the eigenvalues and eigenvectors of the matrix.

$$A = \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix} \quad (10)$$

P.T.O. 