KHYBER PAKHTUNKHWA PUBLIC SERVICE COMMISSION
COMPETITIVE EXAMINATION FOR THE POSTS OF PMS OFFICERS (BPS-17)

PURE MATHEMATICS (PAPER-I)

TIME ALLOWED: THREE HOURS — Max. Marks: 100

Instructions: Attempt three questions from Section A and two questions from Section B.

SECTION A

Q1. (a) Let G be a group of non-zero complex numbers under multiplication and let $N = \{a + ib \in G: a^2 + b^2 = 1\}$ be a subset of G. (10)

(i) Show that N is a subgroup of G.
(ii) Show that $G/N \cong \mathbb{R}^+$, where $\mathbb{R}^+$ is the group of positive real numbers under multiplication.

(b) Let $C_{12} = \{1, b, b^2, \ldots, b^{11}: b^{12} = 1\}$ be a cyclic group of order 12 generated by b. Then, find all the generators of C_{12} other than b. (10)

Q2. (a) Let $S = \{a + ib: a, b \in \mathbb{Z}, b\ is\ even\}$. Show that S is a subring of $\mathbb{Z}[i] = \{r + is: r, s \in \mathbb{Z}\}$ but not an ideal of $\mathbb{Z}[i]$. (10)

(b) Find all $k \in \mathbb{Z}_3$ such that $\mathbb{Z}_3[x] / \langle x^3 + kx^2 + 1\rangle$ is a field. (10)

Q3. (a) Let V be a vector space with $dim(V) = 6$. Suppose U and W are two different vector subspaces of V with $dim(U) = 4 = dim(W)$. Find the possible dimensions of $U \cap W$. (10)

(b) Find the eigenvalues and eigen vectors of the following matrix and diagonalize it if possible (10)

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & 2 \\ 0 & 0 & 2 \end{bmatrix}$$

Q4. (a) For what values of λ the following homogeneous system of linear equations has non-trivial solution. (10)

$$(\lambda + 2)x_1 - 2x_2 + x_3 = 0$$
$$-2x_1 + (\lambda - 1)x_2 + 6x_3 = 0$$
$$x_1 + 2x_2 + \lambda x_3 = 0$$

(b) Find the rank and nullity of $T: \mathbb{R}^3 \to \mathbb{R}^3$ defined by $T(x_1, x_2, x_3) = (x_1 + 2x_2 + 3x_3, 2x_1 + 3x_2 + 4x_3, 3x_1 + 5x_2 + 7x_3)$. (10)

SECTION B

Q5. (a) Find the center, vertices, foci, and asymptotes of the hyperbola $4x^2 - y^2 - 8x - 4y - 4 = 0$. (10)

(b) Find a Cartesian equation for the hyperbola centered at the origin that has a focus at (3, 0) and the line $x = 1$ as the corresponding directrix. (10)

Q6. (a) Find the parametric equations for the lines in which the planes $3x - 6y - 2z = 15$ and $2x + y - 2z = 5$ intersect and find the angle between these two planes. (10)

(b) Write the cartesian equation of $r^2 = 4r\sin(\theta)$ and then graph it. (10)

Q7. (a) Define principal tangent vector $\boldsymbol{T}(t)$, principal normal vector $\boldsymbol{N}(t)$, binormal vector $\boldsymbol{B}(t)$ and curvature $\kappa(t)$. (8)

(b) Let $\boldsymbol{r}(t) = 2\cos t\boldsymbol{i} + 2\sin t\boldsymbol{j} + 3t\boldsymbol{k}$ be a vector function. Find $\boldsymbol{T}(t)$, $\boldsymbol{N}(t)$, $\boldsymbol{B}(t)$ and $\kappa(t)$. (6)

(c) Using vector function given in part (b), find an equation of the osculating plane, the normal plane and the rectifying plane at the point corresponding to $t = \pi/2$. (6)