

KHYBER PAKHTUNKHWA PUBLIC SERVICE COMMISSION
COMPETITIVE EXAMINATION FOR THE POSTS OF PMS OFFICERS (BPS-17)

PURE MATHEMATICS PAPER-II

Time Allowed: 03 Hours

Maximum Marks: 100

Instructions: Attempt three questions from Section A and two questions from Section B.

SECTION A

Q1. (a) Find (i) $\lim_{x \rightarrow 0^+} x^x$ (ii) $\lim_{x \rightarrow \pi/2} (\tan x)^{\cos x}$ (10)

(b) Is there a value of k that will make $f(x) = \begin{cases} x + k, & x < 0 \\ \cos x, & x \geq 0 \end{cases}$ continuous at $x = 0$? Differentiable at $x = 0$? Justify your answer. (10)

Q2. (a) Find the extreme values of the function $f(x, y) = x^3 y^2 (1 - x - y)$. (10)

(b) Evaluate the $\int \frac{dx}{(x^2 - 2x + 1)\sqrt{x^2 - 1}}$. (10)

Q3. (a) Test the convergence of (i) $\sum_{n=1}^{\infty} \frac{n \ln(n)}{2^n}$ (ii) $\sum_{n=1}^{\infty} \frac{(-1)^n \sin(n)}{n^2}$ (10)

(b) Find the value of $\int_0^1 \left(\sum_{n=1}^{\infty} \frac{x^n}{n(n+2)} \right) dx$. (10)

Q4. (a) Show that in a metric space (X, d) , every finite set is closed. (10)

(b) Let (X, d) be a metric space, $A \subseteq X$ and $a \in X$. Show that $a \in \bar{A}$ iff there exists a sequence $\{x_n\}$ in A such that $x_n \rightarrow a$, where \bar{A} stands for closure of set A . (10)

SECTION B

Q5. (a) Find the solution of $\cos(z) = 1/2$. (10)

(b) Show that $\tanh^{-1} z = \frac{1}{2} \log \left(\frac{1+z}{1-z} \right)$. (10)

Q6. (a) Verify that $U(x, y) = \tan^{-1} \left(\frac{y}{x} \right)$ is harmonic in \mathbb{C} and find its conjugate harmonic function. (10)

(b) Find the Laurent series of the function $f(z) = \frac{1}{(z-1)(2-z)}$ in $|z| > 2$. (10)

Q7. (a) Using Residue theorem, find $\int_{|z|=2} \frac{e^z}{z(z-1)^3} dz$. (10)

(b) Let γ be the upper half of a unit circle, oriented counterclockwise. Show that $\left| \int_{\gamma} \frac{e^z}{z} dz \right| \leq \pi e$. (10)